Chapter 8
Topics in Graph Theory

Chapter 8: Topics in Graph Theory

- Section 8.1: Examples {1,2,3}
- Section 8.2: Examples {1,2,4}
- Section 8.3: Examples {1}

8.1 – Graphs

- Graph
  - A graph G consists of a finite set of vertices V, a finite set of edges E, and a function γ that assigns a subset of vertices {v, w} to each edge (v may equal w)
  - If e is an edge, and $\gamma(e) = \{v, w\}$ we say e is the edge between v & w, and that v & w are the endpoints of e

8.1 – Graphs

- Terminology
  - Degree of a vertex
    - Number of edges having that vertex as an endpoint
  - Loop
    - Edge from a vertex to itself
    - Contributes 2 to the degree of a vertex
  - Isolated vertex
    - Vertex with degree 0
  - Adjacent vertices
    - Vertices that share an edge

8.1 – Graphs

- Path
  - A path Π in a graph G consists of a pair of sequences (V_Π, E_Π), V_Π: $v_1, v_2, \ldots, v_k$ and E_Π: $e_1, e_2, \ldots, e_{k-1}$ s.t.:
    1. $\gamma(e_i) = \{v_i, v_{i+1}\}\ \forall i$
    2. $e_i \neq e_j\ \forall\ i, j$

8.1 – Graphs

- More terminology
  - Circuit
    - Path that begins and ends at the same vertex
  - Simple path
    - No vertex appears more than once (except possibly the first and last)
  - Simple circuit
    - Simple path where first and last vertices are equal

8.1 – Graphs

- Special types of graphs
  - Connected graph
    - ∃ path from every vertex to every other (different) vertex
  - Disconnected graph
    - There are at least 2 vertices which do not have a path between them
    - components
  - Regular Graph
    - All vertices have the same degree

8.2
Euler Paths and Circuits

8.2 – Euler Paths and Circuits

- A path in a graph G is an Euler path if it includes every edge exactly once
- An Euler circuit is an Euler path that is also a circuit

8.2 – Euler Paths and Circuits

- Theorem 8.2.1
  1. If G has a vertex of odd degree, there can be no Euler circuit of G
  2. If G is a connected graph and every vertex has even degree then there is an Euler circuit in G
- Theorem 8.2.2
  1. If a graph G has more than 2 vertices of odd degree, there can be no Euler path in G
  2. If G is connected and has exactly 2 vertices of odd degree, then there exists an Euler path in G. Any Euler path must begin at one vertex of odd degree, and end at the other.

8.2 – Euler Paths and Circuits

- Bridge
  - A bridge is an edge in a connected graph that if removed would result in a disconnected graph

Euler circuits and paths

- An *Euler circuit* in a graph is a simple circuit containing every edge of the graph.
- An *Euler path* in a graph is a simple path containing every edge of the graph.
- **Theorem 1**: A finite graph has an Euler circuit if and only if it is connected and all vertices have even degree.

Definitions

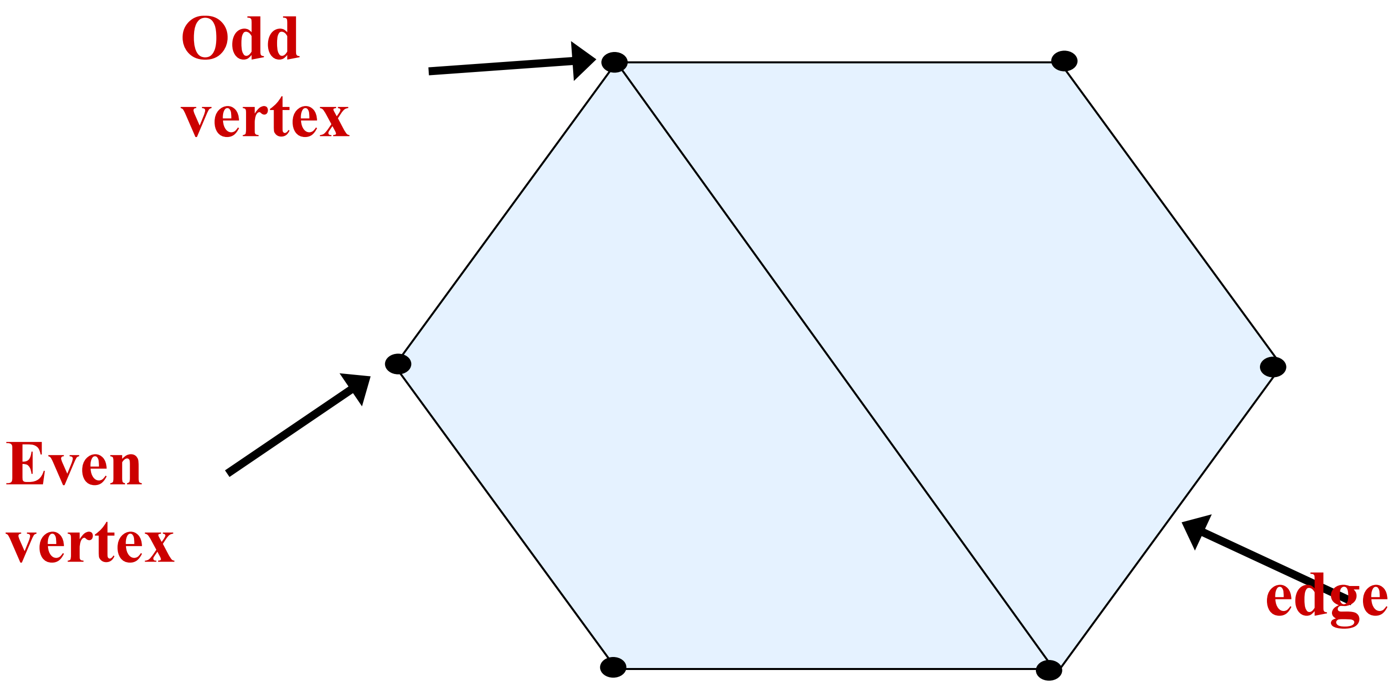
- A **network** ______ is a figure made up of points (vertices) connected by non-intersecting edges

  (Also, known as vertex-edge graphs)

- A **vertex** ________is the **intersection** ______ of two edges.
  - A **vertex** ______ is **odd** ______ if it is connected to an **odd number of edges** ____________
  - A **vertex** ______ is **even** ______ if it is connected to an **even number of edges** ____________

More Definitions

- An __**edge**__ joins any two __**vertices**__. It can be __**curved**__ or __**straight**__.



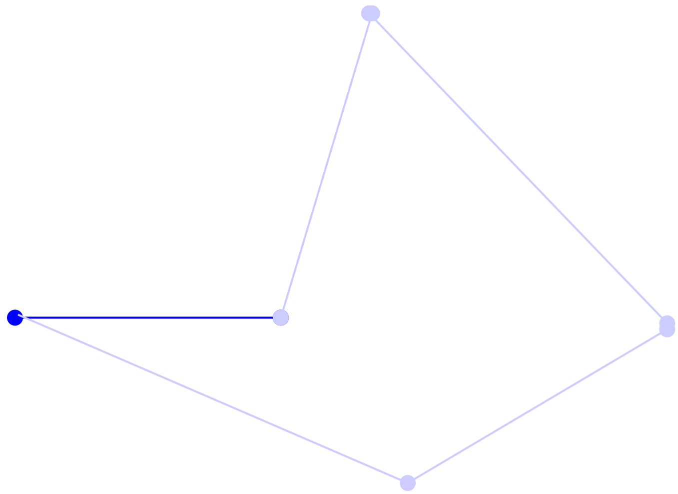
Euler Paths

A graph has an Euler path if it can be **traced in 1 sweep without lifting the pencil from paper AND without tracing the same edge more than once.**

- Vertices may be passed through **more than once.**
- The starting and ending points **are not the same.**

Euler Circuits

A circuit is similar to a Euler path, except **the starting and ending points must be the same.**

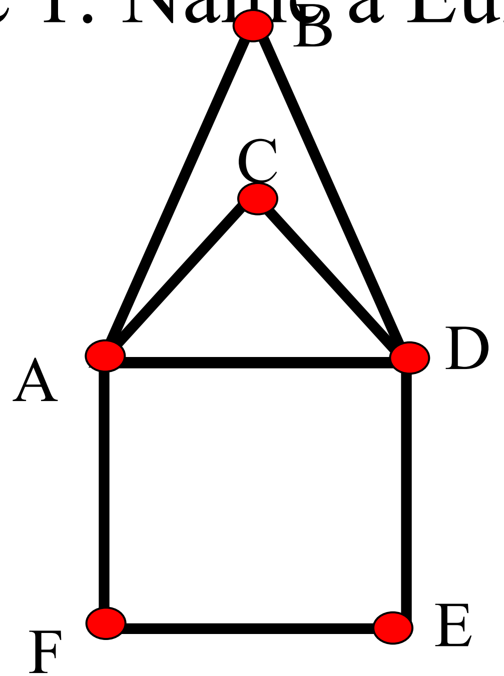


Complete the exploration on the relationship between the nature of the vertices and the kind of graph in your notes.

- Conclusions: Based on the observations of your table:
  - A graph with all vertices being **even** contains an Euler **circuit**____
  - A graph with **2**____ odd vertices and **some even vertices**__ contains an Euler **path**____.
  - A graph with more than 2 **odd**___ vertices does not contain an Euler ____ **path or circuit**

To name a path or circuit you list the vertices in order

Example 1: Name a Euler circuit



One possible solution is

D,E,F,A,D,C,A,B,D

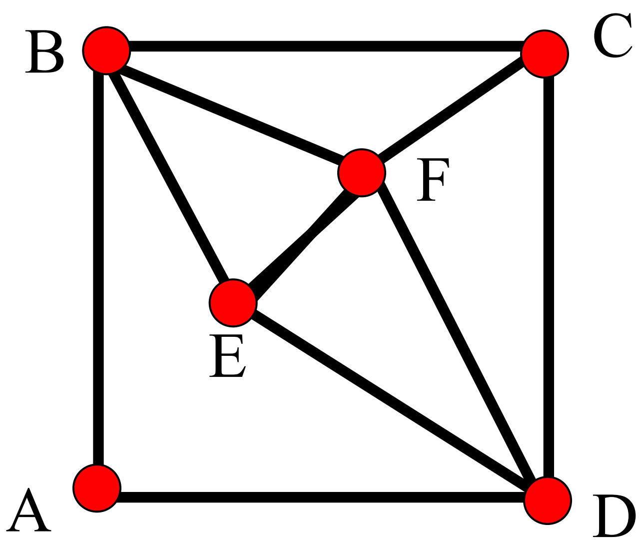
b) Can you find another one?

Example 2:

Given A,B,E,F,B,C,D,F,E,D is this a Euler path or circuit or neither?

How can you tell? Explain your answer

Neither , touches EF twice



Find a Euler circuit if possible, if not list a Euler path

2 odd vertices so has to be a path, starting at E or C

1 Possible solution: EBADEFDCBFC

Hamilton Paths and Circuits

A __Hamilton Path__ is a continuous path that passes through every __vertex__ once and only once.

A __Hamilton Circuit__ is a Hamilton path that begins and ends at the same vertex. (the starting/end vertex will be the only vertex touched twice

How is a Hamilton Path different from a Euler path or Circuit?